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ON AVERAGE AND FINITE PARAMETERS OF THE INTERMITTENT MOTION IN PIN-TYPE ESCAPEMENTS

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Abstract:
The article deals with a certain class of intermittent-motion mechanisms, namely pin-type escapements in regulators without free vibration. A kinematic analysis and displacement diagrams indicate that in practical design calculations it is necessary to consider only the motion (pulse) along the tooth of the ratchet-wheel even though the motion (pulse) along the pin pallet of the pendulum (balance-wheel) may constitute 20% or more of the total motion.

ON AVERAGE AND FINITE PARAMETERS OF THE INTERMITTENT MOTION IN PIN-TYPE ESCAPEMENTS

In the case where the intermittently moving pawls of the pendulum have cylindrical surfaces (pin-type and sharp corners of the pallet), this intermittent motion can be divided into two zones: one along the tooth surface on the wheel and one along the pawl surface or the so-called pulse along the pin.

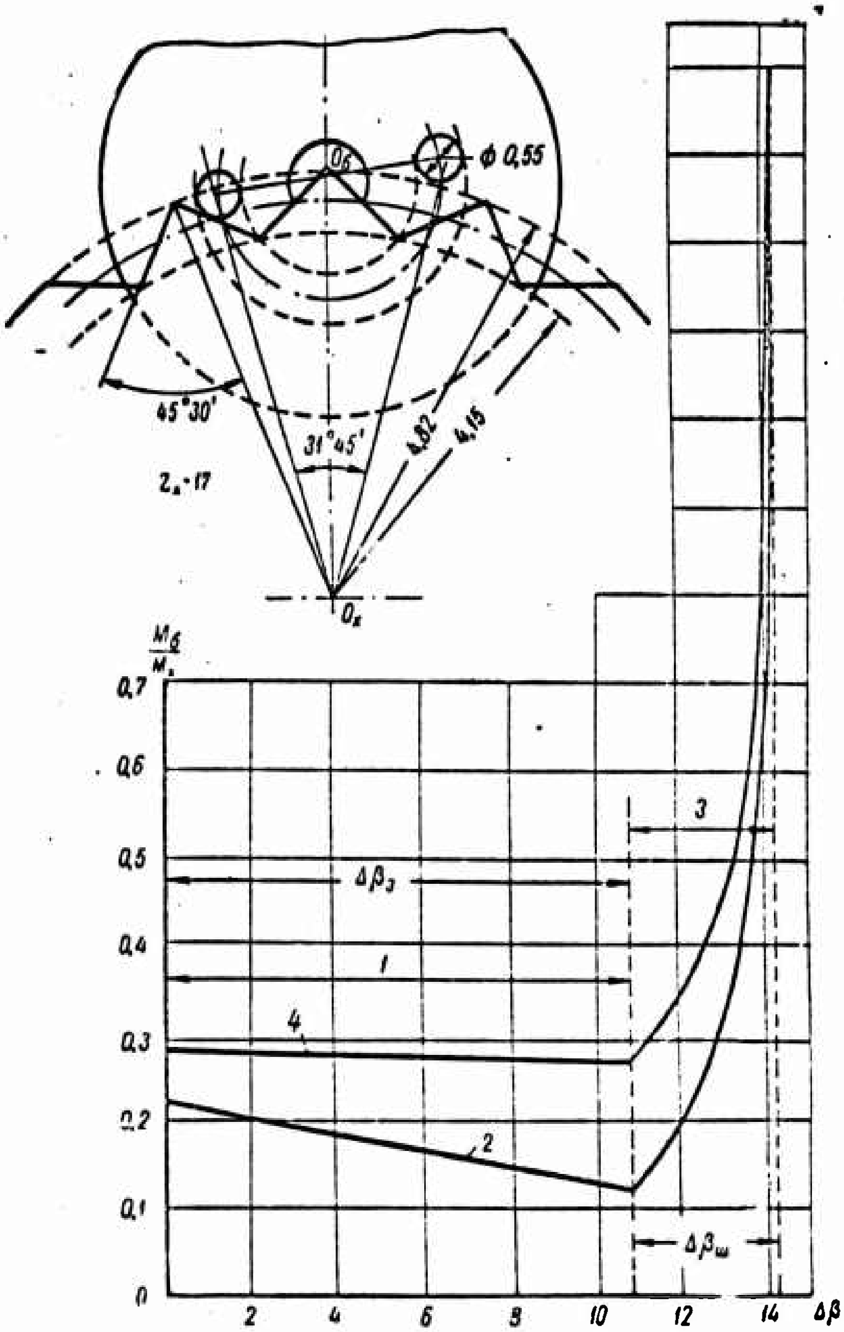
This second portion is quite insignificant in freely anchored pin-type escapements. It may amount to 20% or more of the entire angular sweep, however, when such escapements are part of regulators not oscillating freely and where the pin diameter is, as a rule, over 0.5 mm at a ratchet-wheel diameter of the order of 10 mm and at an angle of embracement up to 1.5 steps, as shown in Fig. 1.

It is well known that, when designing ratchet-type regulators, it is impossible to avoid using average and finite values of the parameters of intermittent motion.

At the same time it is evident from the curves of intermittent displacement in pin-type escapements (Fig. 1) that the average ratio of torques cannot be taken as the arithmetic mean of their initial and final values as is done in the case of other types of escapements; this is because of a clearly indicated overlap between the displacement diagrams within the zone of the pulse along the pin.

It is also impossible to determine the pendulum velocity at the end of the motion on the basis of the ratchet-wheel velocity, since at that instant the wheel/pendulum transmission ratio becomes zero due to the coincidence of the centroid of engagement and the center of the ratchet-wheel.

When the pulse along the pin is very small, it is simply disregarded with the assumption that motion is transmitted along the tooth only. This makes it considerably easier to determine the necessary parameters.



б = b
х = w
з = t
ш = p

Fig. 1. Intermittent-motion curves for a regulator without free vibrations and having pin-type pallets:

1 - motion (pulse) along the tooth (incoming pin);
2 - motion (pulse) along the tooth (outgoing pin);
3 - motion (pulse) along the pin.

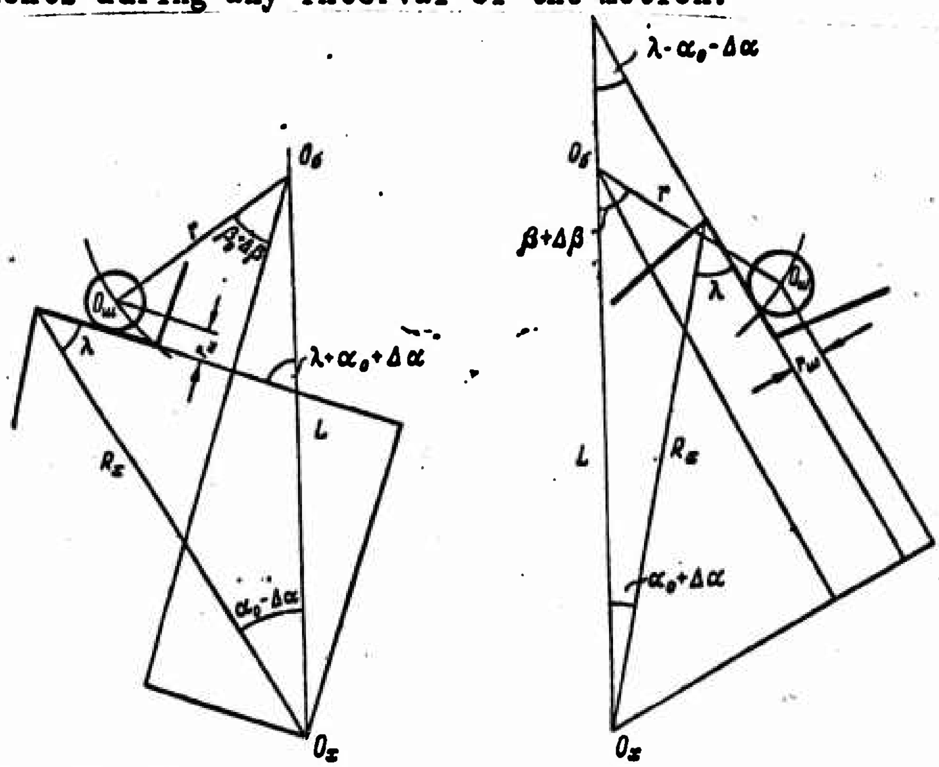
When it becomes evident, however, that the angular displacement along the pin is comparable in magnitude with the angular displacement along the tooth and, therefore, cannot be neglected - the determination of average and finite parameters becomes difficult, especially in the design of regulators without free vibration where an error in the value of torque ratio or transmission ratios during the motion will affect the period of oscillations of the balance wheel.

The following kinematic analysis of pin-type escapements provides a basis for avoiding these aforementioned difficulties.

ANALYSIS OF THE DISPLACEMENT OF RATCHET-WHEEL AND PENDULUM DURING THE INTERMITTENT MOTION

Let us consider the most frequently encountered design of a pin-type escapement with flat tooth surfaces on the ratchet-wheel and let us establish the relation between the angular displacements of the wheel and of the pendulum when there is continuous contact between the tooth and the pin over the entire span of the pulse.

Pulse along the tooth. The angles α_0 and β_0 (Fig. 2) will designate the position of the ratchet-wheel and the pendulum at the beginning of the intermittent motion (these angles can be determined by graphical construction or by calculation), while $\Delta\alpha$ and $\Delta\beta$ are instantaneous angular displacements of the respective escapement components during any interval of the motion.



б = b
ш = p
х = w

Fig. 2. Schematic diagram showing the transmission of intermittent motion along the tooth surface.

In this case at any instant the position of the wheel as viewed from the incoming pallet will be defined by the angle

$$\alpha=\alpha_0-\Delta\alpha,$$

and that of the pendulum by the angle

$$\beta=\beta_0+\Delta\beta.$$

Projecting the escapement dimensions L, R_w, r and r_p on the line perpendicular to the line of motion of the tooth we have

$$L\sin[(\lambda+\alpha_0)-\Delta\alpha]-R_x\sin\lambda-r_{\text{ш}}=r\sin[(\lambda+\alpha_0+\beta_0)-(\Delta\alpha+\Delta\beta)]. \quad (1)$$

Assuming, on account of the smallness of wheel and pendulum displacements, that within the tooth zone of motion

$$\sin\Delta\alpha=\Delta\alpha;\quad \sin\Delta\beta=\Delta\beta;$$

$$\cos\Delta\alpha=1-\frac{(\Delta\alpha)^2}{2};\quad \cos\Delta\beta=1-\frac{(\Delta\beta)^2}{2},$$

we obtain

$$L\sin(\lambda+\alpha_0)-L\frac{(\Delta\alpha)^2}{2}\sin(\lambda+\alpha_0)-L\,\Delta\alpha\cos(\lambda+\alpha_0)-R_x\sin\lambda-r_p=$$

$$=r\sin(\lambda+\alpha_0+\beta_0)-r\frac{(\Delta\alpha)^2}{2}\sin(\lambda+\alpha_0+\beta_0)-r\frac{(\Delta\beta)^2}{2}\sin(\lambda+\alpha_0+\beta_0)+$$

$$+r\frac{(\Delta\alpha)^2\cdot(\Delta\beta)^2}{2}\sin(\lambda+\alpha_0+\beta_0)+r\,\Delta\alpha\Delta\beta\sin(\lambda+\alpha_0+\beta_0)-$$

$$-r\,\Delta\alpha\cos(\lambda+\alpha_0+\beta_0)+r\frac{(\Delta\beta)^2\cdot(\Delta\alpha)^2}{2}\cos(\lambda+\alpha_0+\beta_0)+$$

$$+r\,\Delta\beta\cos(\lambda+\alpha_0+\beta)-r\frac{(\Delta\alpha)^2\cdot(\Delta\beta)^2}{2}\cos(\lambda+\alpha_0+\beta_0).$$

Omitting from this expression the relatively small terms containing products $(\Delta\alpha)^2\cdot(\Delta\beta)^2$; $(\Delta\alpha)^2\Delta\beta$; $\Delta\alpha\,(\Delta\beta)^2$ and

$$L\sin(\lambda+\alpha_0)-R_x\sin\lambda-r_p-r\sin(\lambda+\alpha_0+\beta_0)=0,$$

we can reduce the developed equation to the form

$$(\Delta\beta)^2-2(\Delta\alpha+k_1)\Delta\beta-[(\Delta\alpha)^2\cdot(k_2-1)+2\Delta\alpha(k_3+k_1)]=0, \quad (2)$$

where

$$k_1=\operatorname{ctg}(\lambda+\alpha_0+\beta_0);$$

$$k_2=\frac{L}{r}\frac{\sin(\lambda+\alpha_0)}{\sin(\lambda+\alpha_0+\beta_0)},$$

$$k_3=\frac{L}{r}\frac{\cos(\lambda+\alpha_0)}{\sin(\lambda+\alpha_0+\beta_0)}.$$

Solving the last equation we will obtain an expression which relates the angular displacements of the wheel and the pendulum

$$\Delta\beta = \Delta\alpha + k_1 \pm \sqrt{(\Delta\alpha)^2 k_2 + 2\Delta\alpha k_3 + k_1^2}, \qquad (3)$$

where, in accordance with the condition that

$$\Delta\beta = 0 \quad \text{when} \quad \Delta\alpha = 0$$

the (+) sign applies to $k_1 < 1$ and the (-) sign applies to $k_1 > 1$.

At the outgoing pallet where

$$\alpha = \alpha_0 + \Delta\alpha,$$
$$\beta = \beta_0 + \Delta\beta,$$

we find by analogy

$$\Delta\beta = k_1 - \Delta\alpha \pm \sqrt{k_1^2 - 2\Delta\alpha k_3 - (\Delta\alpha)^2 k_2}, \qquad (4)$$

where

$$k_1 = \operatorname{ctg}(\alpha_0 + \beta_0 - \lambda);$$

$$k_2 = \frac{L}{r}\,\frac{\sin(\lambda - \alpha_0)}{\sin(\alpha_0 + \beta_0 - \lambda)};$$

$$k_3 = \frac{L}{r}\,\frac{\cos(\lambda - \alpha_0)}{\sin(\alpha_0 + \beta_0 - \lambda)}.$$

For $k_1 > 0$ the square root in (4) is taken with the (-) sign, while for $k_1 < 0$ it is taken with the (+) sign. Having derived the formulae which relate the angular displacements, we shall now determine the transmission ratios for the ratchet-wheel and pendulum pair.

Differentiating equations (3) and (4) we obtain for the zone along the tooth (incoming pin)

$$i = \frac{d(\Delta\beta)}{d(\Delta\alpha)} = 1 + \frac{\Delta\alpha k_2 + k_3}{\sqrt{(\Delta\alpha)^2 k_2 + 2\Delta\alpha k_3 + k_1^2}}. \qquad (5)$$

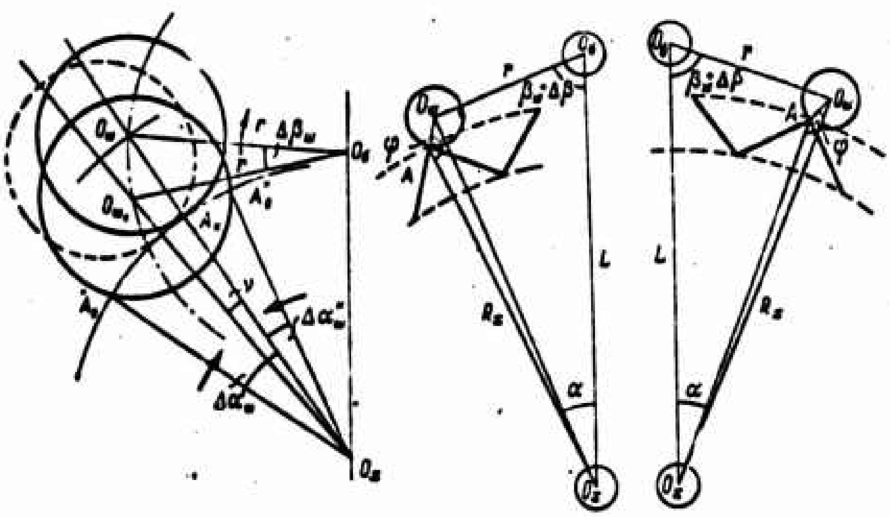
Along the outgoing pin

$$i = \frac{d(\Delta\beta)}{d(\Delta\alpha)} = \frac{\Delta\alpha k_2 + k_3}{\sqrt{k_1^2 - 2\Delta\alpha k_3 - (\Delta\alpha)^2 k_2}} - 1. \qquad (6)$$

Pulse along the pin. As a preliminary step we shall establish how the angles φ depend on the angular displacements of the wheel.

When the incoming and the outgoing pallets are pins and a tooth

tip is located at point A_k at the end of the motion, then the angular displacement of the wheel during the pulse along the incoming pin is larger than that along the outgoing pin $(\Delta\alpha'_p > \Delta\alpha''_p)$ - as can be seen in Fig. 3, where points A'_0 and A''_0 represent the position of tooth tips of the ratchet-wheel at the instant when the pulse along the pin begins[1]. If the pin were not joined to the pendulum but could gradually move into the position indicated in Fig. 3 by the dotted line, then we would obviously have $\Delta\alpha'_p = \Delta\alpha''_p$.



δ = b
ш = p
x = w

Fig. 3. Schematic diagram showing the transmission of intermittent motion along the pin surface.

Actually, however, we have

$$\Delta\alpha'_p - \nu = \Delta\alpha''_p + \nu,$$

wherefrom

$$\nu = \frac{\Delta\alpha'_p - \Delta\alpha''_p}{2}. \qquad (7)$$

The magnitude of angle ν per unit displacement (pulse) angle along the incoming pin is

$$\frac{\nu}{\Delta\alpha'_p} = \frac{\Delta\alpha'_p - \Delta\alpha''_p}{2\Delta\alpha'_p},$$

and referred to the outgoing pin it is

$$\frac{\nu}{\Delta\alpha''_p} = \frac{\Delta\alpha'_p - \Delta\alpha''_p}{2\Delta\alpha''_p}.$$

It is evident from Fig. 3 that

1) Superscripts ' and " designate angles referring to the incoming and the outgoing pallet.

$$\varphi = \pi - (\angle AO_wO_p + \angle AO_pO_w).$$

However

$$\angle AO_wO_p = \Delta\alpha_p - \Delta\alpha \mp \frac{v}{\Delta\alpha_p}(\Delta\alpha_p - \Delta\alpha),$$

where the (-) sign applies to the incoming pallet and the (+) sign applies to the outgoing pallet.

Designating the constant coefficient as

$$a = 1 \mp \frac{\Delta\alpha'_p - \Delta\alpha''_p}{2\Delta\alpha_w},$$

we have

$$\angle A O_wO_p = (\Delta\alpha_p - \Delta\alpha)\, a.$$

Considering now that the angles AO_wO_p are small which permits them to be equated with their sines, the Law of Sines for triangle O_wAO_p yields

$$\angle AO_pO_x = \arc\sin \frac{R_w}{r_p}(\Delta\alpha_p - \Delta\alpha)\, a.$$

Thus

$$\varphi = \pi - U, \tag{8}$$

where the following designation has been made:

$$U = (\Delta\alpha_p - \Delta\alpha)\, a + \arcsin \frac{R_w}{r_p}(\Delta\alpha_p - \Delta\alpha)\, a.$$

Returning again to Fig. 3 we find from triangles O_wAO_p and $O_wO_pO_b$ by equating the expressions for their common side O_wO_p that

$$\cos(\beta_p + \Delta\beta) = \frac{L^2 + r^2 - R_w^2 - r_p^2 + 2R_wr_p\cos\varphi}{2Lr}.$$

Substituting the values for angle φ and letting

$$L^2 + r^2 - R_w^2 - r_p^2 = K,$$

we arrive at an equation which relates the angular displacements of the ratchet-wheel and the balance-wheel in the form

$$\Delta\beta = \arccos \frac{K - 2R_wr_p\cos U}{2Lr}, \tag{9}$$

where

$$U = f(\Delta\alpha).$$

For obtuse angles this equation becomes

$$\Delta\beta = \pi - \beta_p - \arccos\frac{K - 2R_W r_p \cos U}{2Lr}. \qquad (9a)$$

The equation for the transmission ratio in the zones of motion along the pin is obtained by differentiating equation (9) for the incoming and for the outgoing pin and has the form:

$$i = \frac{d(\Delta\beta)}{d(\Delta\alpha)} = \frac{R_W r_p}{Lr} \cdot \frac{a\left[1 + \dfrac{R_W}{r_p\sqrt{1 - \dfrac{R_W}{r_p} a(\Delta\alpha_p - \Delta\alpha)}}\right]\sin U}{\sqrt{1 - \left(\dfrac{K - 2R_W r_p \cos U}{2Lr}\right)^2}}. \qquad (10)$$

CONCLUSIONS

The changes of angular displacement and transmission ratios are plotted graphically in Fig. 4, 5 as functions of the ratchet-wheel rotation and calculated according to formulae (3), (4), (5), (9) and (10) applicable to the pin-type regulator escapement without free vibrations as shown in Fig. 1.

The curves $\Delta\beta = f(\Delta\alpha)$ obtained here indicate that, with flat engaging tooth surfaces, the angular displacements of the pendulum (balance-wheel) along the major part of the motion may be considered proportional to the angular displacements of the ratchet-wheel. In the zone along the tooth it is permissible, therefore, to use simplified relations between angular displacements of the ratchet-wheel and the pendulum for practical design calculations:

$$\Delta\beta = b \cdot \Delta\alpha. \qquad (11)$$

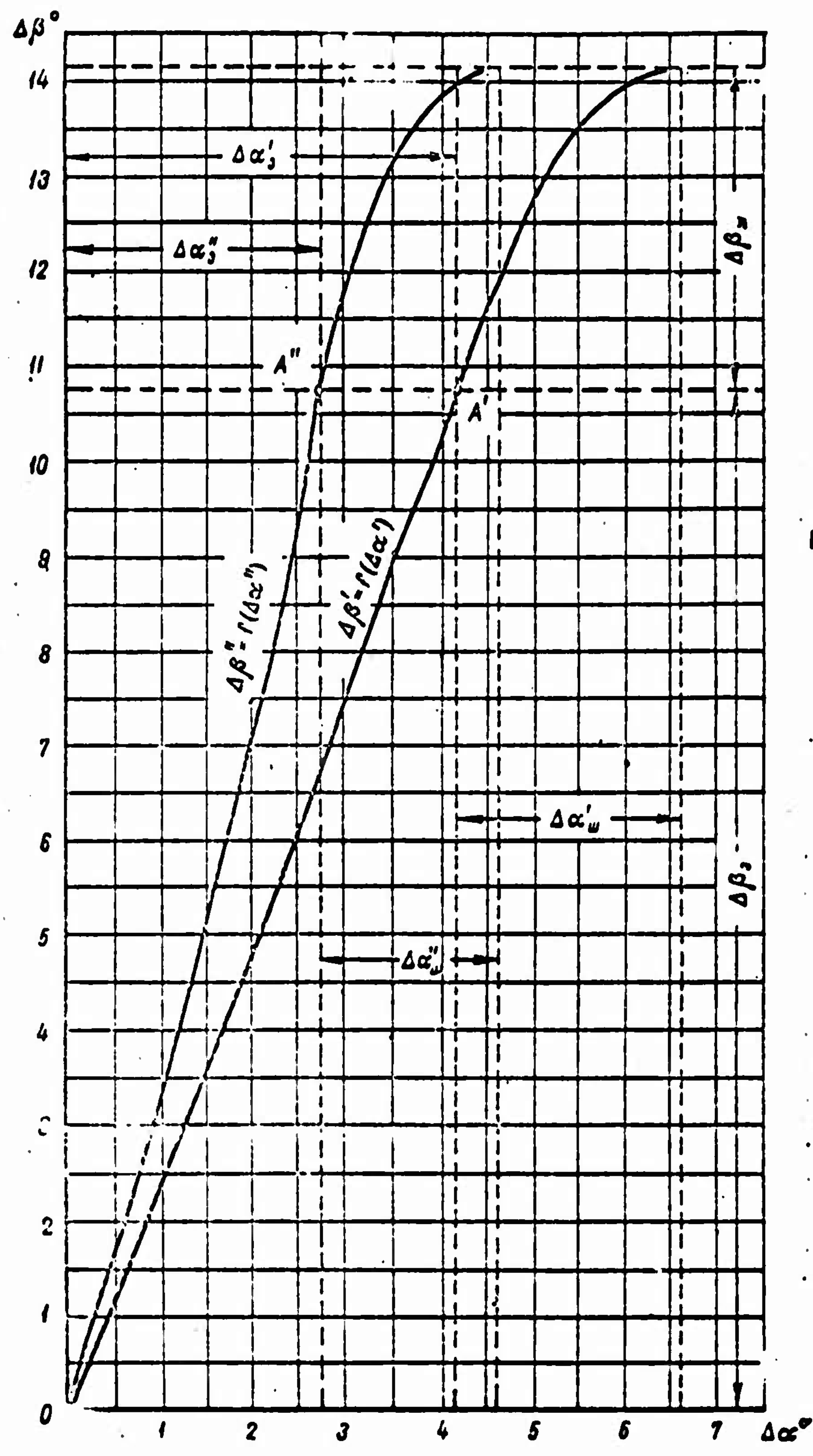
If the angular displacements of the pendulum and of the ratchet-wheel during the pulse along the tooth are designated by $\Delta\beta_t$ and $\Delta\alpha_t$, then

$$b = \frac{\Delta\beta_t}{\Delta\alpha_t}, \qquad (12)$$

where $\Delta\beta_t$ and $\Delta\alpha_t$ can be found by plotting the displacement diagram.

The difference in transmission ratio variations along the tooth zone and the pin zone is distinctly indicated by the graph in Fig. 5 Within the first zone this transmission ratio wheel-to-pendulum increases but at a relatively slow rate; within the second zone it decreases fast and reaches a zero value at the end of the motion.

In accordance with the link-mechanism theory, such a characteristic $i = f(\Delta\alpha)$ means that the accelerations in the escapement are for both members positive during the pulse along the tooth and are of opposite signs during the pulse along the pin.



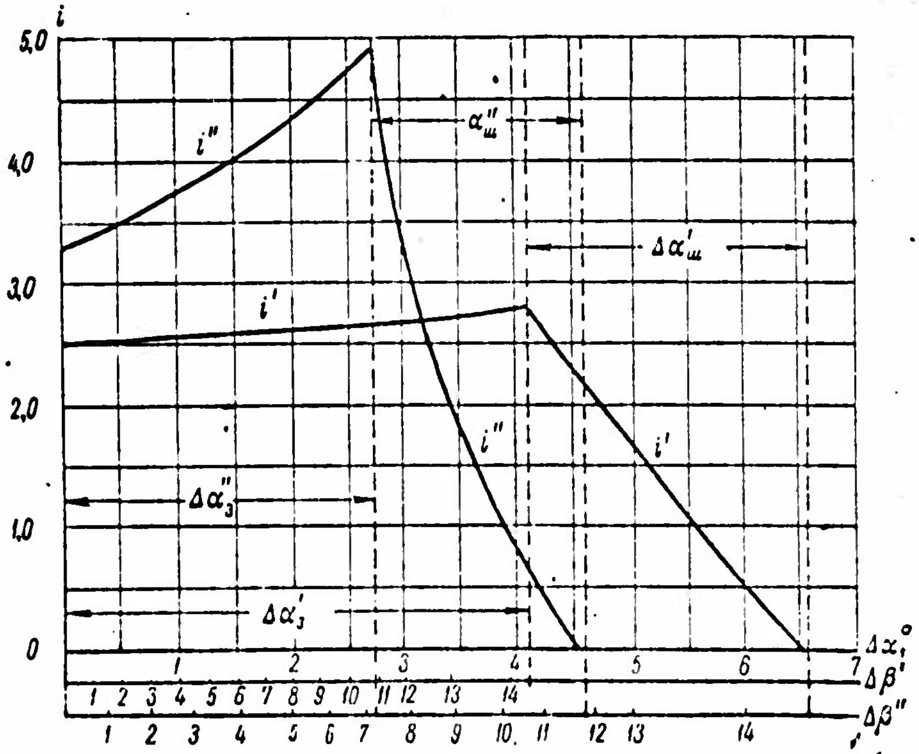
з = t
ш = p

Fig. 4. Curves representing the angular displacement of the pendulum (balance-wheel) as a function of the angular displacement of the ratchet-wheel.

However, a ratchet-wheel driven by a continuous torque cannot have negative acceleration. Consequently, only the pendulum (balance-wheel) will be decelerated within the second zone of motion.

This leads to the conclusion that there is a discontinuity in the acceleration diagram of the balance-wheel at the point where the tooth edge passes the pin.

The presence of a discontinuity confirms the occurrence of soft impulses during the motion, as a result of which the force acting on the pin varies instantaneously in magnitude between finite limits [1].



ш = p
3 = t

Fig. 5. Curves representing the transmission ratio between ratchet-wheel and pendulum in a pin-type escapement.

Thus, the accelerated motion of the ratchet-wheel not-withstanding, its contact with the pin during the pulse along the pin if not lost completely is nevertheless loosened abruptly, and the pendulum or balance-wheel joined to that pin will move faster when free than when subject to the torque.

In order to simplify the calculations connected with the determination of average and finite parameters of motion in pin-type escapements, therefore, it is necessary to take into account only zones of motion along the tooth surfaces of the ratchet-wheel.

Calculations similar to those shown here were made also for other regulator escapements without free vibrations. Their results are analogous to those obtained for the escapement in Fig. 1 and they confirm the validity of the conclusion arrived at in this article.

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